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Fixed Point Theorems using Sub-Compatibility and Sub-Sequentially Continuity in Fuzzy Metric Spaces

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Abstract - We acquire fixed point hypotheses through sub compatible and subsequentially continues for four self-guides in fluffy measurement spaces. Our outcome generalizes numerous known results and investigates the chance of applying the design of sub-compatible and subsequentially consistent to the issue of finding fixed purposes of four mappings or succession of mappings fulfilling happenstance point conditions containing levelheaded terms in fluffy measurement spaces.
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1. INTRODUCTION

In 1965 Zadeh [12] presented the thought of fluffy sets. After this numerous creator have set up the presence of a loads of preset point hypotheses in fluffy settings Badard [1], Bose and Sahani [2], Fang [3], Hadzic [6], Heilpern [7], Kaleva [8]. Later numerous creators have widely built up the hypothesis of fluffy sets and application. The possibility of fluffy measurement space presented [9] was adjusted by Veeramani [4]. [10] presented the thought of Semi-viable guides in fluffy measurement space, and acquired fixed point hypotheses in complete fluffy measurement space in the feeling of Grabiec [5]. In [11] Vasuki presented the idea of R-pitifully driving guide and demonstrated fixed point hypotheses in fluffy measurement space. In this element we build up certain outcomes utilizing diverse similarity alongside the utilization of successive congruity and verifiable relations in fluffy measurement space.

2. TERMS AND NOTATIONS

In this part we review a few definitions and known outcomes in fluffy measurement space.

Definition 2.1 [12]: The 3-tuple (X, M, *) is identified as a fluffy measurement space (quickly, FM-space) if X is a self-assertive set * is a continious t-standard and M is a fluffy set in $X^2x[0,1)$ fulfilling the accompanying conditions:

(FM - 1) M(x, y, 0) = 0
(FM - 2) M(x, y, 0) = 1, for all t>0 iff x = y
(FM - 3) M(x, y, t) = M(y,x,t),
(FM - 4) M(x, y, t) * M(y, z, s) ≤ M(x, z, t+s) and
(FM - 5) M(x, y, .): [0,1) - > [0, 1] is left persistent for all x,y,z has a place with X and s, t>0
(FM - 6) limt→∞ M(x, y, t) = 1 for $\forall x \in X$ and t>0.

Definition 2.2: Compatible guides: Let f and g act naturally maps on a fluffy measurement space (X, M, *).
They are viable or asymptotically driving if $\forall t > 0$,

$$\lim_{n\to\infty} M\left(fgx_n, gfx_n, t\right) = 1$$

Whenever $\{x_n\}$ is a sequece X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$, $\exists\ z \in X$.

Mappings f and g are noncompatible maps, if ∃ $\{x_n\}$ in X: $\lim_{n\to\infty} fx_n = p = \lim_{n\to\infty} gx_n$ but either $\lim_{n\to\infty} M(fgx_n, gfx_n) \neq 1$ or the limit $\neg\exists(\forall(p \in X))$.

Definition 2.3 Compatible guides type (I): Let f and g act naturally maps on a fluffy measurement space (X, M, *). A couple {f, g} is supposed to be:
(f) Compatible of type (I) if for ∀ t > 0,

$$\lim_{n\to\infty} M(fgx_n, gfx_n) \leq \mathrm{M(gx, x, t)}$$

Whenever $\{x_n\}$ is a sequence in $X : \lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$, $\exists\ x \in X$
(g) Compatible of type (II) if the pair (g,f) is compatible of type (I).

Definition 2.4 Coincidence focuses: Let X be a set, f, g selfmaps of X. A point x in X is known as a fortuitous event purpose of f and g if fx = gx. We will call w=fx=gx a state of happenstance of f and g.

Definition 2.5 Weakly Compatible guides: The mappings f and g from a fluffy measurement space (X, M, *) into itself are pitifully viable in the event that they drive at their happenstance point, that is fx = gx infers that gfx = fgx. It is realized that a couple {f, g} of viable guides is feebly viable yet banter isn't accurate as a rule. This can be found in following model.

Example 2.1: Let (X, M, *) be a fuzzy metric space, where X = [0,2], t-norm is defined by a*b = min{a,b}, ∀ $a, b \in [0,1]$ and $M(\mathrm{x,y,t}) = \mathrm{e}^{\frac{-|x-y|}{t}}$ for all $x, y \in X$ and all t>0. Define self maps A and S on X as follows:

$$Ax = \begin{cases} 2-x & if\ 0 \leq x \leq 1 \\ 2 & if\ 1 \leq x \leq 2 \end{cases} \qquad Sx = \begin{cases} x & if\ 0 \leq x \leq 1 \\ 2 & if\ 1 \leq x \leq 2 \end{cases}$$

Take $x_n = 1 - \frac{1}{n}$ then $x_n \to 1, x_n < 1$ and $2-x_n>1$. Also $Ax_n, Sx_n \to 1$ as $n \to \infty$

Now $M(\mathrm{ASx}_n, \mathrm{SAx}_n, \mathrm{t}) = \mathrm{e}^{\frac{-|ASx_n - SAx_n|}{t}} \to e^0 = 1$

as $n \to \infty$. So A and S are not compatible. The set of coincident points of A and S is [1,2]. For any $x \in [1,2]$, $Ax = Sx = 2$ and ASx = A(2) = 2= S(2) =Sax. Thus A and S are weak-compatible but not compatible.

3. Examples

Following model shows that the class of engrossing guides is neither a sub class of viable guides nor a sub class of non-viable guides

Example 1. *Let (X, d) be usual metric space where X = [2, 20] and M be the usual fuzzy metric on (X, M, *) where * = t_{min} be the induced fuzzy metric space with* $M(\mathrm{x,y,t}) = \frac{t}{t + d(\mathrm{x,y})}$ *and M(x,y,0)= 0 for* $x, y \in X,\ t > 0$ *.we define mappings A, B, S and T by*

$$fx = 6\ \ if\ \ 2 \leq x \leq 5, f6 = 6, fx = 10\ \ if\ \ x > 6,$$

$$fx = \frac{x-1}{2}\ if\ x \in (5,6), \mathrm{gx} = 2\ \ if\ \ 2 \leq x \leq 5,\ \ gx = \frac{(\mathrm{x}+1)}{3}\ \ if\ \ x > 5,$$

It is easy to see that both pairs (f, g) and (g, f) are not compatible but f is g -absorbing and g is f - absorbing.

Example 2. *Let X = [0,1] be a metric space and ○ and , are same as in example 1. Define A, B:* $X \to X$ *by* $Ax = \frac{x}{16}$ *and* $Bx = 1 - \frac{x}{3}$,

In this example we can see that

(1) A and B are compatible pair of maps and

(2) An is B - retaining while B is A – engrossing Here scope of A = [0, 1/16] and scope of B = [2/3, 1]. In the following model it is indicated that engrossing guides need not drive at their incident focuses, hence the idea of retaining maps is not the same as different speculations of commutativity which power the planning to drive at fortuitous event focuses.

Example 3. *Let X = [0, 1] be a metric freedom and d and M are identical as in pattern 1.*
Define $f, g : X \to X$ *by*

$$fx = 1 \ \ for\ x \neq 1,\ f1 = 0\ and\ gx = 1\ for\ x \in X$$

Then the maps f is g - engrossing for any R > 1 but the pair of maps (f, g) do not travel at their luck point x = 0.

Definition 3.1 ***Semi-compatible*:** A pair (A,S) of self-maps of a fuzzy metric space (X,M, *) is said to be *semi-compatible* if $\lim_{n\to\infty} Ax_n = Sx$ at any time $\{x_n\}$ is a sequence such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x \in X$$

It follows that if (A,S) is semi-compatible and Ay = Sy then ASy = SAy.

Remark 3.1: Let (A,S) be a duo of self-maps of a fuzzy metric freedom (X, M, *) Then (A,S) is *R*-weakly commuting implies that (A, S) is well-suited, which implies that (A,S) is weak-compatible. But the converse is not true.

4. Main Results

Theorem 4.1: Let A, B, S and T be four self maps of fuzzy metric break (X, M, *) with continuous t-norm * defined by $t * t \geq t$ for all $t \in [0,1]$. If the pairs (A,S) and (B,T) are subcompatible and subsequentially continuous, then

a) A and S have an occurrence point.

b) B and T have an occurrence point.

c) $M(\mathrm{Ax,By,Kt}) \geq \min\{\mathrm{M(Sx,Ty,t), M(Ax,Ty,t), M(By,Sx,2t) * M(Sx,Ty,t)}, \frac{M(\mathrm{Ax,Ty,t}) * \mathrm{M(By,Ty,t)}}{M(\mathrm{Ax,Sx,t})}\}$

Then A, B, S, T have exclusive fixed point.

Proof:- Since pair (A,s) and (B,T) are subcompatible and subsequentially continuous then there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z,\ z \in X$$ And satisfy

$$M(\mathrm{ASx}_n, \mathrm{SAx}_n, \mathrm{t}) = \mathrm{M(Az, Sz, t)} = 1$$

$$\lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z',\ z' \in X$$

Which satisfy

$$M(\mathrm{BTy}_n, \mathrm{TBy}_n, \mathrm{t}) = \mathrm{M(Bz', Tz', t)} = 1$$

$$\therefore Az = Sz \ \text{ and } \ Bz' = Tz';$$

i.e z is a luck point of A and S and z' is a luck point of B and T. Now to prove z = z'.

Put x= x_n and y = y_n in inequality (c), we get

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$$M(Ax_n, By_n, Kt)$$
$$\geq \min\{M(Sx_n, Ty_n, t), M(Ax_n, Ty_n, t), M(By_n, Sx_n, 2t)$$
$$*M(Sx_n, Ty_n, t), \frac{M(Ax_n, Ty_n, t) * M(By_n, Ty_n, t)}{M(Ax_n, Sx_n, t)}\}$$

Taking limit $n \to \infty$ we get

$$M(z, z', Kt) \geq \min\{M(z, z', t), M(z, z't), M(z, z', 2t) * M(z, z', t), \frac{M(z, z', t) * M(z', z', t)}{M(z, z, t)}\}$$

$M(z, z', Kt) >= \{M(z, z', t)\}$ [By lemma(3.1.1)]

$z = z'$

Again we claim that Az = z. Substitute x = z and y = y_n in inequality we have

$$M(Az, By_n, Kt)$$
$$\geq \min\{M(Sz, Ty_n, t), M(Az, Ty_n, t), M(By_n, Sx_n, 2t)$$
$$*M(Sz, Ty_n, t), \frac{M(Az, Ty_n, t) * M(By, Ty_n, t)}{M(Az, Sz, t)}\}$$

As limit $n \to \infty$

$$M(Az, z', Kt)$$
$$\geq \min\{M(Sz, z', t), M(Az, z', t), M(z', Sz, 2t)$$
$$*M(Sz, z', t), \frac{M(Az, Ty_n, t) * M(By, Ty_n, t)}{M(Az, Sz, t)}\}$$

$$M(Az, z', Kt)$$
$$\geq \min\{M(Az, z', t), M(Az, z', t), M(z', Az, 2t)$$
$$*M(Az, z', t), \frac{M(Az, Ty_n, t) * M(By, Ty_n, t)}{M(Az, Sz, t)}\}$$

$$M(Az, z', Kt) \geq \{M(Az, z', t)\}$$
$$\therefore Az = z' = z = Sz$$

Again we claim that Bz = z. Substitute x =z and y = z in inequality we have

$$M(Az, Bz, Kt)$$
$$\geq \min\{M(Sz, Tz, t), M(Az, Tz, t), M(Bz, Sz, 2t)$$
$$*M(Sz, Tz, t), \frac{M(Az, Tz, t) * M(Bz, Tz, t)}{M(Az, Sz, t)}\}$$

$$M(\mathrm{z},\mathrm{Bz},\mathrm{Kt})$$
$$\geq \min\{M(\mathrm{z},\mathrm{Bz},\mathrm{t}), M(\mathrm{z},\mathrm{Bz},\mathrm{t}), M(\mathrm{Bz},\mathrm{z},2\,\mathrm{t})$$
$$*M(\mathrm{z},\mathrm{Bz},\mathrm{t}), \frac{M(\mathrm{z},\mathrm{Bz},\mathrm{t}) * M(\mathrm{Bz},\mathrm{Bz},\mathrm{t})}{M(\mathrm{z},\mathrm{z},\mathrm{t})}\}$$
$$M(\mathrm{z},\mathrm{Bz},\mathrm{Kt}) \geq M(\mathrm{z},\mathrm{Bz},\mathrm{t})$$
$\therefore z = Bz = Tz$

Uniqueness: - Let w be an additional general fixed peak of A, B, S, T then
“Aw = Bw = Sw = Tw = w”
Place x = z and y = w in inequality, we get

$$M(\mathrm{Az},\mathrm{Bw},\mathrm{Kt})$$
$$\geq \min\{M(\mathrm{Sz},\mathrm{Tw},\mathrm{t}), M(\mathrm{Az},\mathrm{Tw},\mathrm{t}), M(\mathrm{Bz},\mathrm{Sz},2\,\mathrm{t})$$
$$*M(\mathrm{Sz},\mathrm{Tw},\mathrm{t}), \frac{M(\mathrm{Az},\mathrm{Tw},\mathrm{t}) * M(\mathrm{Bw},\mathrm{Tw},\mathrm{t})}{M(\mathrm{Az},\mathrm{Sz},\mathrm{t})}\}$$
$$M(\mathrm{z},\mathrm{w},\mathrm{Kt})$$
$$\geq \min\{M(\mathrm{z},\mathrm{w},\mathrm{t}), M(\mathrm{z},\mathrm{w},\mathrm{t}), M(\mathrm{w},\mathrm{z},2\,\mathrm{t})$$
$$*M(\mathrm{z},\mathrm{w},\mathrm{t}), \frac{M(\mathrm{z},\mathrm{w},\mathrm{t}) * M(\mathrm{w},\mathrm{w},\mathrm{t})}{M(\mathrm{z},\mathrm{z},\mathrm{t})}\}$$
$$M(\mathrm{z},\mathrm{w},\mathrm{Kt}) \geq M(\mathrm{z},\mathrm{w},\mathrm{t})$$
$\therefore z = w$

Therefore, uniqueness follows.

Theorem 4.2: Let A, B, S and T be four self maps of fuzzy metric gap (X, M, *) with constant t-norm * defined by t * t >= t, $\forall\ t \in [0,1]$. If t pairs (A,S) and (B,T) are subcompatible and subsequentially continuous, then

a) A and S have a happenstance point.
b) B and T have a happenstance point.
c)
$$M(\mathrm{Ax},\mathrm{By},\mathrm{Kt})$$
$$\geq \min\{M(\mathrm{Sx},\mathrm{Ty},\mathrm{t}), M(\mathrm{Ax},\mathrm{Ty},\mathrm{t}), M(\mathrm{By},\mathrm{Sx},2\,\mathrm{t})$$
$$*M(\mathrm{Sx},\mathrm{Ty},\mathrm{t}), \frac{M(\mathrm{Ax},\mathrm{Ty},\mathrm{t}) * M(\mathrm{By},\mathrm{Ty},\mathrm{t})}{M(\mathrm{Ax},\mathrm{Sx},\mathrm{t})}\}$$

Then A, B, S, T have exclusive fixed point.

Proof: Theorem can easily be proved in the lines of Theorem 3.3.1.

If we put A=B and S=T in 3.3.2 we get.

Corollary 4.1: Let A, S be two nature maps of wooly metric freedom (X, M, *) with unbroken t norm * defined by t * t >= t for all $t \in [0,1]$. If the pair (A, S) is subcompatible and subsequentially permanent, then

a)A and S have a happenstance summit.

b)
$$M(\mathrm{Ax},\mathrm{Ay},\mathrm{Kt}) \geq \min\{M(\mathrm{Sx},\mathrm{Sy},\mathrm{t}), M(\mathrm{Ax},\mathrm{Sy},\mathrm{t}), M(\mathrm{Ay},\mathrm{Sx},2\,\mathrm{t}) * M(\mathrm{Sx},\mathrm{Sy},\mathrm{t}), \frac{M(\mathrm{Ax},\mathrm{Sy},\mathrm{t}) * M(\mathrm{Ay},\mathrm{Sy},\mathrm{t})}{M(\mathrm{Ax},\mathrm{Sx},\mathrm{t})}\}$$

Then A and S have exclusive fixed point.

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