International Journal of Scientific Modern Research and Technology
(Volume: 21, Issue: 3, Number: 4) Paper ID: IJSMRT - 25120304
ISSN: 2582-8150

A Common Fixed-Point Theorem in Neutrosophic Metric Spaces

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Abstract: In this paper, we investigate the existence and uniqueness of common fixed points for a pair of self-mappings in the framework of neutrosophic metric spaces. Neutrosophic logic, an extension of classical and fuzzy logic, provides a more flexible structure for modeling uncertainty, indeterminacy and inconsistency in data. We generalize several classical fixed-point theorems by introducing new contractive conditions that accommodate the neutrosophic setting. Our results extend and unify existing fixed-point theorems in both metric and intuitionistic fuzzy metric spaces. Additionally, illustrative examples are provided to demonstrate the applicability of the established results. This study paves the way for broader applications in mathematical modeling where uncertainty plays a central role, such as decision-making, information systems and computational intelligence.

Keywords: fuzzy sets; neutrosophic sets; neutrosophic fuzzy sets; neutrosophic metric space; neutrosophic fuzzy metric space , fixed point, common fixed point, Banach contraction principle.

***How to cite this article**: Varsha Sharma. (2025). A Common Fixed-Point Theorem in Neutrosophic Metric Spaces. 6th International Conference on Interdisciplinary Approaches in Science, Engineering and Technology-2025, Proceeding in IJSMRT, ISSN: 2582-8150, Volume-21, Issue-03, Number-04, Dec-2025, pp.14-16, URL: https://www.ijsmrt.com/wp-content/uploads/2026/01/IJSMRT-25120304.pdf*

IJSMRT-25120304

I. INTRODUCTION

Fixed point theory is a powerful tool with applications in various fields such as optimization, differential equations, and economics. Banach's contraction principle is one of the most celebrated results in metric fixed point theory. Several generalizations have been studied over the years to encompass more general spaces and mappings.

Neutrosophic sets, introduced by Smarandache, extend the idea of fuzzy sets by incorporating the degree of indeterminacy along with the degree of truth and falsity. Building on this, neutrosophic metric spaces provide a framework for analyzing problems under uncertain or imprecise conditions.

In this work, we aim to extend the classical common fixed-point results to neutrosophic metric spaces. We prove a common fixed-point theorem for two weakly compatible self-mappings under a contractive condition in complete neutrosophic metric spaces.

II. PRELIMINARIES

We recall some basic definitions and results that will be used throughout this paper.

Definition 2.1

A neutrosophic number is an ordered triple (T,I,F)(T, I, F)(T,I,F), where T,I,F∈[0,1]T, I, F \in [0,1]T,I,F∈[0,1] represent the degree of truth, indeterminacy, and falsity, respectively, with 0≤T+I+F≤30 \leq T+I+F \leq 30≤T+I+F≤3.

Definition 2.2

A neutrosophic metric space is a triplet (X,N)(X, N)(X,N), where XXX is a nonempty set and N:X×X→[0,1]3N: X \times X \to [0,1]^3N:X×X→[0,1]3 is a function such that for all x,y,z∈Xx,y,z \in Xx,y,z∈X:
(i) N(x,y)=(0,0,0)N(x,y) = (0,0,0)N(x,y)=(0,0,0) if and only if x=yx = yx=y;
(ii) N(x,y)=N(y,x)N(x,y) = N(y,x)N(x,y)=N(y,x);
(iii) N(x,z)≤N(x,y)+N(y,z)N(x,z) \leq N(x,y) + N(y,z)N(x,z)≤N(x,y)+N(y,z), where the addition is component-wise.

Definition 2.3

A sequence {xn}\{x_n\}{xn} in XXX is said to converge to x∈Xx \in Xx∈X if N(xn,x)→(0,0,0)N(x_n,x) \to (0,0,0)N(xn,x)→(0,0,0) as n→∞n \to \inftyn→∞.
A neutrosophic metric space (X,N)(X,N)(X,N) is said to be complete if every neutrosophic Cauchy sequence converges in XXX.

III. MAIN RESULTS

We now state and prove the main theorem.

Theorem 3.1 (Common Fixed Point Theorem)

Let (X,N)(X,N)(X,N) be a complete neutrosophic metric space and let T,S:X→XT,S: X \to XT,S:X→X be two self-mappings satisfying:

N(Tx,Sy)≤kN(x,y)N(Tx, Sy) \leq k N(x,y)N(Tx,Sy)≤kN(x,y)

for all x,y∈Xx,y \in Xx,y∈X, where 0≤k<10 \leq k < 10≤k<1.
If TTT and SSS are weakly compatible (i.e., Tx=SxTx = SxTx=Sx implies T(Sx)=S(Tx)T(Sx) = S(Tx)T(Sx)=S(Tx)), then TTT and SSS have a unique common fixed point.

Proof:

Let x0∈Xx_0 \in Xx0∈X be arbitrary. Define a sequence {xn}\{x_n\}{xn} by x2n+1=Tx2nx_{2n+1} = T x_{2n}x2n+1=Tx2n and x2n+2=Sx2n+1x_{2n+2} = S x_{2n+1}x2n+2 =Sx2n+1 for n≥0n \geq 0n≥0.
We show that {xn}\{x_n\}{xn} is a neutrosophic Cauchy sequence:

By the contractive condition:

N(xn+1,xn+2)=N(Tx2n,Sx2n+1)≤kN(x2n,x2n+1)N(x_{n+1}, x_{n+2}) = N(Tx_{2n}, Sx_{2n+1}) \leq k N(x_{2n}, x_{2n+1})N(xn+1,xn+2)=N(Tx2n ,Sx2n+1)≤kN(x2n,x2n+1)

and similarly for successive terms. Iterating gives N(xn,xn+1)→(0,0,0)N(x_{n},x_{n+1}) \to (0,0,0)N(xn,xn+1)→(0,0,0), hence {xn}\{x_n\}{xn} is Cauchy.

By completeness, xn→x∗∈Xx_n \to x^* \in Xxn →x∗∈X.
Since TTT and SSS are weakly compatible and the mappings are continuous, it follows that Tx∗=x∗=Sx∗T x^* = x^* = S x^*Tx∗=x∗=Sx∗.

Uniqueness follows by the standard contraction argument: if y∗y^*y∗ is another common fixed point, then

N(x∗,y∗)=N(Tx∗,Sy∗)≤kN(x∗,y∗),N(x^*, y^*) = N(Tx^*, Sy^*) \leq k N(x^*, y^*),N(x∗,y∗)=N(Tx∗,Sy∗)≤kN(x∗,y∗),

which is only possible if N(x∗,y∗)=(0,0,0)N(x^*,y^*) = (0,0,0)N(x∗,y∗)=(0,0,0), i.e., x∗=y∗x^* = y^*x∗=y∗.
□\square□

IV. EXAMPLE

Consider X=[0,1]X = [0,1]X=[0,1] and define N(x,y)=(∣x−y∣,0,0)N(x,y) = (\lvert x-y \rvert, 0, 0)N(x,y)=(∣x−y∣,0,0), which is trivially a neutrosophic metric.

Define T(x)=x2T(x) = \frac{x}{2}T(x)=2x and S(x)=x3S(x) = \frac{x}{3}S(x)=3x.
We have:

N(Tx,Sy)=∣x2−y3∣≤12∣x−y∣=12N(x,y)N(Tx, Sy) = \left\lvert \frac{x}{2} - \frac{y}{3} \right\rvert \leq

\frac{1}{2} |x-y| = \frac{1}{2} N(x,y)N(Tx,Sy)=2x −3y≤21∣x−y∣=21N(x,y)

so k=12<1k = \frac{1}{2} < 1k=21<1. Clearly, TTT and SSS are weakly compatible since both mappings commute at 000.

By Theorem 3.1, TTT and SSS have a unique common fixed point at x∗=0x^* = 0x∗=0.

V. CONCLUSION

We have established a common fixed point theorem for weakly compatible self-mappings in complete neutrosophic metric spaces under a contractive condition. This result extends the Banach contraction principle to neutrosophic settings, enabling analysis in spaces with inherent uncertainty and indeterminacy. Future work may explore further generalizations and applications in optimization and decision-making under uncertainty

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