

# Design and Analysis of Nano Vibratory Beam Gyroscope

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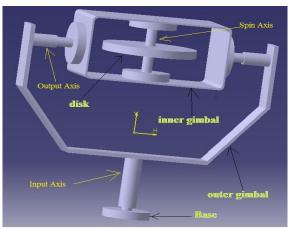
Abstract- Gyroscope is an angular rate measurement sensor having broad application in the field of automotive, military services, aerospace and consumer electronics industries. Silicon micro machined MEMS vibratory gyroscopes have better advantages compared to conventional gyroscope. Nano beam vibratory gyroscope is one of the simple gyroscope. It has relatively small size, light weight, low power consumption, low cost and simple structure. When a gyroscope is made to rotate at its base along with some excitation in one of the bending direction, due to Coriolis Effect, there will be significant displacement in other bending direction. Dynamic modeling of beam gyroscope is very interesting area. In the micro/nano level actuation and sensing are with electrostatics principles. This report presents the modeling and analytical simulation task of a nano cantilever beam gyroscope. Static and dynamic analysis of a nano/micro cantilever gyroscope with a tip mass is studied. Pull-in instability corresponding voltage is estimated from static and frequency response. Pull-in stability regions are identified as a function of beam length, tip mass value, elastic modulus of the beam. Nonlinearities due to geometry and the external forces including electrostatic and Vander Waals forces are considered during modelling. Squeeze film and slide film damping are considered to account the damping force between the tip mass and sense and drive direction. The dynamic solution is obtained by using Galerkin's reduction scheme. The time response and the frequency domain graphs are arrived for different parameters on both sense and drive directions. The interactive program developed in the work is helpful to account any experiments for additional force at nano level. Keywords: MEMS, Nano beam, Gyroscope

### 1. INTRODUCTION

Micro machined angular rate gyroscopes are often finding applications in several systems including aviation, consumer electronics and defense sectors. Various MEMS gyroscopes are being recently developed and implemented in various applications. Design and analysis, fabrication and electronic circuitry are typical issues in development of MEMS/NEMS gyroscope. Vibratory MEMS gyroscopes convert the mechanical displacements into equivalent electrical voltage in sense direction. Conversion efficiency (sensitivity), operating range, accuracy of measurements (resolution) are ultimate parameters for gyroscope designation. Tuning fork type, beam type and ring type are few commonly used vibratory gyroscopes.

Gyroscope first discovered in 1817, by Johann Bohnenberger. Gyroscopes are the angular rate sensors which can be used for measuring or maintaining orientation from the principles of angular momentum.

The device is having a disk or wheel and an inner gimbal and an outer gimbal as shown in Fig. 1.1.



#### Figure 1: Traditional Gyroscope

The axle of disk is free to take any of the orientation. The gimbals are used to minimize the external torque. Traditional gyroscope having a disk which is rotating along the direction of the engine shaft. The axis of rotation of the disk is called spin axis. Rotating disk is



surrounded by an inner gimbal. Inner gimbal can also rotate about its axis. The axis of inner gimbal is perpendicular to the spin axis and inner gimbal is covered by an outer gimbal whose axis is perpendicular to inner gimbal axis. The spin axis, inner gimbal axis and outer gimbal axis are mutually perpendicular to each other. One of inner or outer gimbal axis represents the input axis and other one as output axis. When a small disturbance comes in input axis then the combined effect of spinning and disturbance will observed in output axis which is known as gyroscopic effect. The output axis also known as precession axis.

## 2. LITERATURE REVIEW

**Katz and Highsmith [2020]** studied about the optimal size of the vibratory beam type gyroscope because the thermal noise is depend upon the beam length. They concluded that for longer beam the thermal noise is lower. These work very helpful for the application of gyroscope in aviation because noise play important role for the development of aviation vehicles.

Yang and Fang [2019] performed the vibration study of elastic beam having piezoelectric surface bonded films and rotating about one of its axis. They also considered the effect of centripetal force and Coriolis forces. They proposed a beam model can be used for gyroscope.

**Seok and Scarton [2018]** studied the dynamic characteristics of a beam type angular rate gyroscopic sensor. They considered the square cross-section of the vibratory beam and performed the sensitivity and bandwidth analysis of these beam. They concluded that by increasing the bandwidth of the sensor the sensitivity decreases.

**Esmaeili et al. [2004]** study the performance and dynamic modelling of a vibratory beam type gyroscope by considering general support motion. They considered that the beam vibrate in all 3 direction and the beam is rotate about longitudinal direction. Equation of motion are derived by using Extended Hamilton Principle. They considered the effect of Coriolis accelerations, angular accelerations, and beam distributed mass, centripetal accelerations and tip mass on the performance of gyroscope.

Ashokanthan and Cho [2003] investigated the dynamic stability of beam type gyroscope under the rate fluctuations. For fluctuations in velocity of rotating

beam type gyroscope a mathematical model is developed. The system is having gyroscopic coupling so due to these gyroscopic coupling the variation in natural frequency are characterized. The dynamic stability is investigated due to variation in input angular speed. Numerical integration technique is used to validate the results.

**Bhadbhade et al. [2001]** studied about the vibrating beam type gyroscope which is having a cantilever beam fixed at one end and a tip mass is attached to its other end and it is piezo-electrically actuated. Extended Hamilton principle used for mathematical modelling of the system. Results shows that the performance of gyroscope is depend upon the secondary base rotation of the beam. They also concluded that with increase in beam length, primary excitation amplitude and base rotation rate the gyroscopic effect will increase.

# 3. PROBLEM IDENTIFICATION

- Beam is the distribute parameter system with two bending displacement. When such a system is made to rotate at its base along with some excitation in one of the bending direction
- Difficult to apply this method due to high current density and ease of control the electrostatic actuation.
- Its base along with some excitation in one of the bending direction, due to Coriolis Effect there will be significant displacement in other bending direction.

## 4. RESEARCH OBJECTIVES

The objectives of this work are:

- Mathematical modeling of the beam type gyroscope and derive the general equation of motion for the system.Static analysis task by eliminating the time coordinate from the equation and considering the effect of intermolecular force.
- Reduction of 4th order partial differential equation into 2nd orders ordinary differential equation by using Galerkin's technique and solves the equations by R-K method.
- Dynamic analysis of the beam by considering the effect of inters molecular forces and damping forces. Construct the lumped



parameter model for the system to get dynamic response.

# 5. METHODOLGY

Beam is the distribute parameter system with two bending displacement. When such a system is made to rotate at its base along with some excitation in one of the bending direction, due to Coriolis Effect there will be significant displacement in other bending direction. This chapter presents dynamic equations of motion of beam gyroscope in lumped parameter spring mass form, distributed continuous system form and also numerical finite element form. The solution of these equations is shown as static, frequency and dynamic analysis outputs. Approximate and numerical techniques are used to solve the differential equations.

Actuation techniques are very important for design and development of micro gyroscope. There are many actuation technique used for analysis point of view of micro and nano gyroscope, but electrostatic actuation, magnetic and piezoelectric actuations are the conventional method which are generally used for micro gyroscope. Due to high current density and ease of control the electrostatic actuation technique is most widely used actuation technique now days. In this work electrostatic technique is used for both driving and sensing purpose.

In electrostatic technique two conductors are used. Voltage is supplied between these conductors so due to potential difference the electric field is generated by the charge particle. The electrostatic field applied by the generated electric field. The force expression can be derived by differentiating the energy stored per unit length in the capacitor with respect to gap between the conductors.

The electrostatic force is inversely proportional to the square of the gap between the conductors so this is the main problem of electrostatic actuation but in microscopic scale it is beneficial because the micro structures has low aspect ratio and the gap between the conductor also very small. In the design of MEMS system this is the most widely used technique compare to other one. It is also used in accelerometer, switches, micro resonators and micro mirrors etc.

### 6. RESULT AND DISCUSSION

The effect of damping and inter molecular forces considered in the dynamic equation and the results for

combined effect obtained. The deflection and velocity response in sense direction with combined effect of damping and inter molecular forces. Similarly, The deflection and velocity response in drive direction with the combined effect of damping and inter molecular forces. The results shows that without damping forces the amplitude of vibration increasing continuously in case of both with and without inter molecular forces, but when we consider the damping for both the cases then the amplitude of vibration first increases and after some cycle it remain constant throughout the time domain. There is significant variation observed by considering the effects of inter molecular forces. The amplitude of vibration differs for both the cases.

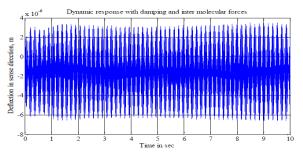


Figure 2: Time Verses Deflection Curve in Sense Direction with Inter Molecular and Damping Forces by Using Galerkin's Technique

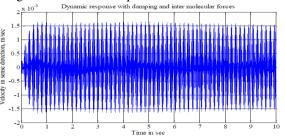


Figure 3: Time Verses Velocity Curve in Sense Direction with Inter Molecular and Damping Forces by Using Galerkin's Technique

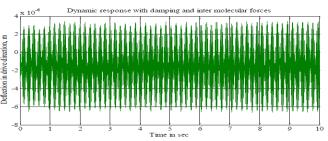


Figure 4: Time Verses Deflection Curve in Drive Direction with Inter Molecular and Damping Forces by Using Galerkin's Technique

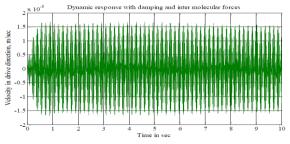


Figure 5: Time Verses Velocity Curve in Drive Direction with Inter Molecular and Damping Forces by Using Galerkin's Technique

#### 7. CONCLUSIONS AND FUTURE SCOPE

This work presented the design issues of nano-beam gyroscope, and its static and dynamic analysis results. The governing equations of motion were obtained by Hamilton's principle and validated from literature. The rotation of the beam was considered to be about longitudinal (x) axis, and the beam has square crosssection which is uniform throughout the length of the beam. A tip mass is attached to its end. Electrostatic sensing and actuation principles are used. The static pull-in voltage was derived at different parameter such as different rotation, different beam lengths and widths of the beam, different electrode areas, gap between the electrode and tip mass and different values of density. Inter molecular forces considered are vander waal and Casimir forces which are highly nonlinear function of displacements and the static pull-in curve drawn and the comparison made between the static pull-in behaviors of with and without inter molecular forces. Following inferences can be written:

- Pull-in voltage decreases with increase of input angular rotation.
- Pull-in voltage value decreases by increase in length of the beam.
- By increasing the width of beam the pull-in voltage also increase.
- When the area of the drive and sense electrode increases then pull-in voltage decreases.
- Pull-in voltage increases by incrementing the gap between the tip mass and electrode.
- With the increasing of density of the material the pull-in voltage also increases.

The natural frequency analysis also performed. The variation of the fundamental natural frequency verses applied DC voltage curves are drawn for different tip mass ratio. The dynamic analysis also carried out. The time verses deflection and velocity curves plotted for lumped parameter model of the nano vibratory beam gyroscope for drive and sense direction. The 4th order partial differential equation is reduced to 2nd order ordinary differential equation with the help of Galerkin's decomposition technique and the 2nd order differential equation in time variable is solved by Runge-Kutta method. The tip displacements histories were obtained by considering damping and inter molecular forces. It was found that there is a marked difference, if these forces are not considered. An attempt was made to obtain the micro cantilever sample for conducting further experiments.

The silicon nano cantilever beam may be fabricated and the electrostatic forces can be generated by certain mechanisms. The set up can be tested using laser Doppler vibrometer for the dynamic response. Further, the results of the analysis require validation with FE method proposed in this work. Also, optimized dimensions of beam may be arrived by maximizing the sensitivity of the gyroscope. The range and resolution of this gyroscope needs to be specified by considering an electronic circuitry.

#### REFERENCES

[1] D. M. Shupe and J.M.O. Connor, 'Vibratory beam rotating sensor', US patent, 4, pp 381-672, 1983.

[2] K. Tanaka, Y. Mochida, M. Sugimoto, K. Moriya, T. Hasegawa, K. Atsuchi and K. Ohwada, 'A micromachined vibratory gyroscope', Sensor and actuator A, Vol. 50, pp 111-115,1995.

[3] K. Maenaka, T. Fujita, Y. Konishi and M. Maeda, 'Analysis of highly sensitive silicon gyroscope with cantilever vibrating mass', Sensor and actuator A, Vol. 54, pp 568-573, 1996.

[4] Katz and A. Highsmith, 'The optimal size of a resonant vibrating beam Gyroscope', Journal of Dynamic Systems, Measurement, and Control Vol. 123, pp 49–54, 2001.

[5] J. Yang and H. Fang, 'Analysis of a Rotating Elastic Beam with Piezoelectric Films as an Angular Rate Sensor', IEEE Transactions on Ultrasonics,



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ferroelectrics, and frequency control, Vol. 49, pp 798-804, 2002.

[6] S. Kausinis and R. Barauskas, 'Computer simulation of piezoelectric angular rate sensor', Measurement, vol. 39, pp.947-958, 2006.

[7] Qingkai Yu, Guoting Qin, Chinmay Darne, Chengzhi Cai , Wanda Wosik and Shin Shem Pei, 'Fabrication of short and thin silicon cantilevers for AFM with SOI wafers', Sensors and Actuators, Vol. 126, pp 369–374, 2006.

[8] J.Seoka and H. Scarton, 'Dynamic characteristics of a beam angular-rate sensor', International Journal of Mechanical Sciences, Vol. 48, pp. 11–20, 2006.

[9] M. Esmaeili, N. Jalilib and M. Durali, 'Dynamic modeling and performance evaluation of a vibrating beam microgyroscope under general support motion', Journal of Sound and Vibration Vol. 301, pp. 146-164, 2007.

[10] F. Asokanthan, and J. Cho, 'Dynamic Stability of Beam-type Vibratory Angular Rate Sensors Subjected to Rate Fluctuations', J. Int.Material systems and Structures, Vol. 19, pp. 735-743, 2008.

[11] V. Bhadbhade, N. Jalili and S. Mahmoodi, 'A novel piezoelectrically actuated flexural/torsional Vibrating beam gyroscope', Journal of Sound and Vibration, Vol. 311, pp. 1305–1324, 2008.

[12] M. Ghommem, A. Nayfeh, S. Choura, F. Najar and E. Abdel-Rahman, 'Modeling and performance study of a beam microgyroscope', Sound and Vibration, Vol. 329, pp. 4970-4979, 2010.