Higher Modes Natural Frequencies of Stepped Beam using Spectral Finite Element

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Abstract- The dynamic behavior of a structure is of great importance in engineering for which it is necessary to accurately predict the dynamic characteristics of the structure. The finite element method (FEM) has been used extensively in structural dynamics. The finite element model may provide accurate dynamic characteristics of a structure if the wavelength is large compared to the mesh size. However, the finite element solutions become increasingly inaccurate as the frequency increases. Although the accuracy can be improved by refining the mesh, this is sometimes prohibitively expensive. The conventional finite element (mass and stiffness) matrices are usually formulated from assumed frequency- independent polynomial shape functions. Because the vibrating shape of a structure varies with the frequency of vibration in reality, the FEM requires subdivision of the structure into finite elements (or a mesh) for accurate solutions. Alternatively, if the shape functions are frequency dependent, then the subdivision may not be necessary. The spectral element method gives frequency dependent dynamic element stiffness matrix regard less of the length or size of the element. Once this Stiffness Matrix for an element is formulated the global Dynamic Stiffness Matrix is obtained by following the procedure similar to that of the Finite Element Method (FEM). The great advantage of such a matrix is that even higher frequencies of a structure can be obtained by considering only few elements thus minimizing the computational cost. In this thesis the higher mode natural frequencies of a stepped beam are obtained. The natural frequency of a stepped beam was found up to the tenth mode by just considering two spectral elements.
Keyword: Ultimate Bearing Capacity, Reinforced Sand Bed, Eccentric Loading

I. INTRODUCTION

One of the fundamental characteristics of the wave propagation problem is that the incident pulse duration is very small (of the order of micro seconds) and hence the frequency content of pulse is very high (of the order of kHz). When such a pulse is applied to the structure, it will force all the higher order modes to participate in the response. At higher frequencies, the wave lengths are small. Hence, in order to capture all the higher order modes, the conventional finite element method requires very fine mesh to match the wavelengths. This makes the system size enormously large. The spectral element approach (SEA) could be the nice alternative for such problems .In SEA, first the governing equation is transformed in frequency domain using discrete Fourier transform (DFT). In doing so, for 1D waveguides, the governing partial differential equation (PDE) is reduced to a set of ordinary differential equations (ODE) with constant coefficients, with frequency as a parameter. The resulting ODEs are much easier to solve than the original PDE. The SEA begins with the use of exact solution to governing ODEs in the frequency domain as interpolating function. The use of exact solution results in exact mass distribution and hence the resulting dynamic stiffness matrix is exact. Hence, in the absence of any discontinuity, one single element is sufficient to handle a beam of any length. This substantially reduces the system size and they are many orders smaller than the sizes involved in the conventional FEM. First, the exact dynamic stiffness is used to determine the system transfer function (frequency response function). This is then convolved with load. Next, inverse fast Fourier transform (IFFT) is used to get the time history of the response.

In the wave propagation problems, as the frequency of the input loading is very high, the short term effects are critical. To get the accurate mode shapes and natural frequencies, the wave length and mesh size should be small. Alternatively we can use the time marching schemes under the finite element environment. In this method, analysis is performed over a small time step, which is a fraction of total time for which response histories are required. For some time marching schemes, a constraint is placed on the time step, and this, coupled with very large mesh sizes, make the solution of wave propagation problem. Wave propagation deals with loading of very high frequency content and finite element (FE) formulation for such problems is computationally prohibitive as it requires large system size to capture all the higher modes.

II. PREVIOUS WORK

Anthony Patera et al [2020] The spectral element method is a high-order finite element technique that combines the geometric flexibility of finite elements with the high accuracy of spectral methods. This method was pioneered in the mid 1980's It exhibits several favorable computational properties, such as the use of tensor products, naturally diagonal mass matrices, and adequacy to implementations in a parallel computer system. Due to these advantages, the spectral element method is a viable alternative to currently popular methods such as finite volumes and finite elements, if accurate solutions of regular problems are sought.

Narayanan and Beskos [2019] introduced the fundamental concept of SEM for the first time. He derived an exact dynamic stiffness matrix for the beam element and employed FFT for dynamic analysis of plane frame-works.Spectral analysis was usually used in fluid dynamic problems, aerospace engineering etc.

Abdelhmid and McConnell [2019] introduced idea of spectral analysis for non-stationary field measurements. Published his first work on the formulation of the spectral element for the longitudinal wave propagation of rods. The term spectral element method in structural dynamics in his work for the DFT/FFT-based spectral element analysis approach. A comprehensive list of the works by Doyle's research group and other researcher's up to 1997.

Doyle and Farris[2018] There he describes about spectral analysis of wave motion and presented an FFT-based Spectral Analysis Methodology, also explains about longitudinal waves and flexural waves in rods and beams respectively. Here spectral element formulation for bars beams and plates are also shown. Presented spectral formulation of finite element for flexural wave propagation in beams.

Banerjee and Williams [2017] presented an elegant and efficient alternative procedure for calculating the number of clamped-clamped natural frequencies of the bending-torsion coupled beam exceeded by any trial frequency, thus enabling the Wittrick-Williams algorithm to be applied with ease when finding the natural frequencies of structure which incorporate such members.

III. PROBLEM IDENTIFICATION

1. The conventional finite element (mass and stiffness) matrices are usually formulated from assumed frequency- independent polynomial shape functions.

2. The vibrating shape of a structure varies with the frequency of vibration in reality the FEM requires subdivision of the structure into finite elements (or a mesh) for accurate solutions.

3. The shape functions are frequency dependent, and then the subdivision may not be necessary.

IV. RESEARCH OBJECTIVES

1. The spectral element method gives frequency dependent dynamic element stiffness matrix regard less of the length or size of the element.

2. To obtain the higher mode natural frequencies of a stepped beam.

3. To develop the empirical correlation for bearing capacity of eccentrically loaded footings on reinforced send by knowing the bearing capacity of footing under centric load.

V. METHODOLOGY

The major significance of this element is that it treats the mass distribution exactly and there for wave propagation within each element is treated exactly. It also means that the subdivision of the member into many small elements is no longer necessary. Consider a rod of length L, where u (x, t) the displacement in the x direction .where ρA is the mass density per unit length of volume.

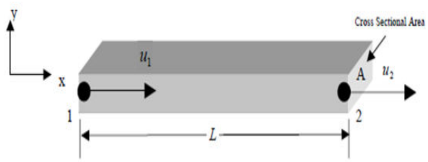

Figure 1: Standard Bar Element

Consider the equations of motion of rod without neglecting the inertia. And assume that there are no applied loads between the rod ends. The general solution for the rod can be represented as:-

$$u(x,t) = \sum_n \hat{u}_n \ (x, \omega_n) e^{-i\omega_n t}$$

Assume that both modulus E and area A do not vary with position, and then the homogenous differential equation for the Fourier coefficients becomes

$$EA \frac{d^2 \hat{u}}{dx^2} + \omega^2 \rho A \hat{u} = 0$$

Where the spectral displacement u^n have the simple solution

$$\hat{u}_n \ (x) = Ae^{-ik_n x} + Be^{-ik_n(L-x)} \quad , \ k_n = \omega_n \sqrt{\frac{\rho A}{EA}}$$

In finite element terms, this is called shape function, but obviously in this case it is dependent on frequency That is, it is different at each frequency unlike the shape function in finite element terms. The nodal displacement can be related to the coefficient by imposing that

$$\hat{u}(0) = \hat{u}_1 = A + Be^{-ikL}, \quad \hat{u}(L) = \hat{u}_2 = Ae^{-ikL} + B$$

i.e. $$\begin{Bmatrix} \hat{u}_1 \\ \hat{u}_2 \end{Bmatrix} = \begin{bmatrix} 1 & e^{-ikL} \\ e^{-ikL} & 1 \end{bmatrix} \begin{Bmatrix} A \\ B \end{Bmatrix}$$

$$\begin{Bmatrix} A \\ B \end{Bmatrix} = \frac{1}{(1-e^{-i2kL})} \begin{bmatrix} 1 & -e^{-ikL} \\ -e^{-ikL} & 1 \end{bmatrix} \begin{Bmatrix} \hat{u}_1 \\ \hat{u}_2 \end{Bmatrix}$$

Allowing the displacement distribution to be written in terms of nodal values as

$$\hat{u}(x) = \frac{1}{(1-e^{-i2kL})} \left[\left(e^{-ikx} - e^{-ik(2L-x)}\right)\hat{u}_1 + \left(e^{-ik(L-x)} - e^{-ik(L+x)}\right)\hat{u}_2 \right]$$

The axial force at arbitrary position is related to nodal displacements by

$$\hat{F}(x) = EA \frac{\partial u}{\partial x}$$

$$\hat{F}(x) = \frac{EA}{L} \frac{iLk}{(1-e^{-i2kL})} \left[\left(-e^{-ikx} - e^{-ik(2L-x)}\right)\hat{u}_1 + \left(e^{-ik(L-x)} + e^{-ik(L+x)}\right)\hat{u}_2 \right]$$

Since nodal forces are related to member forces by $\hat{F}_1 = -\hat{F}(0)$, $\hat{F}_2 = \hat{F}(L)$ then , in matrix notation it can be expressed in the form.

$$\begin{Bmatrix} \hat{F}_1 \\ \hat{F}_2 \end{Bmatrix} = \frac{EA}{L} \frac{iLk}{(1-e^{-i2kL})} \begin{bmatrix} 1+e^{-i2kL} & -2e^{-ikL} \\ -2e^{-ikL} & 1+e^{-i2kL} \end{bmatrix} \begin{Bmatrix} \hat{u}_1 \\ \hat{u}_2 \end{Bmatrix}$$

This can be written in the familiar form of $\hat{F} = [\hat{k}]\{\hat{u}\}$ where $[\hat{k}]$ is the frequency dependent dynamic element stiffness for the rod. It is symmetric and real. This can be confirmed by expanding above to trigonometric expression

$$\begin{Bmatrix} \hat{F}_1 \\ \hat{F}_2 \end{Bmatrix} = \frac{EA}{L} \frac{kL}{\sin kL} \begin{bmatrix} \cos kL & -1 \\ -1 & \cos kL \end{bmatrix} \begin{Bmatrix} \hat{u}_1 \\ \hat{u}_2 \end{Bmatrix}$$

VI. RESULTS

The ultimate bearing capacity of reinforced sand for both cases i.e. B/L=0.33 and 0.5 with different values of *e/B* and *N* has been tabulated in Table 5.7 and Table 5.8.

$$R_{KR} = \alpha_1 \left(\frac{d_f}{B}\right)^{\alpha_2} \left(\frac{e}{B}\right)^{\alpha_3}$$

Where a1, a2, a3 are dimensionless constants. The purpose of the present study is to find out the coefficient a1,a2a3 for rectangular footing by conducting a number of laboratory model tests using rectangular footing with B/L=0.5 & 0.33 resting over multi-layered geogrid reinforced sand bed.

ANALYSIS OF RECTANGULAR FOOTING WITH B/L=0.5

$\frac{B}{L}$	$\frac{d_f}{B}$	$\frac{e}{B}$	$q_{uR(e)}$ (kN/m²)	$\frac{q_{uR(e)}}{q_{uR(e=0)}}$	$R_{KR} = 1 - \frac{q_{uR(e)}}{q_{uR(e=0)}}$
0.5	0.6	0.05	198	0.90	0.10
0.5	0.6	0.10	165	0.75	0.25
0.5	0.6	0.15	132	0.60	0.40

Table 1: Experimental reduction factor for eccentrically loaded footing resting on reinforced

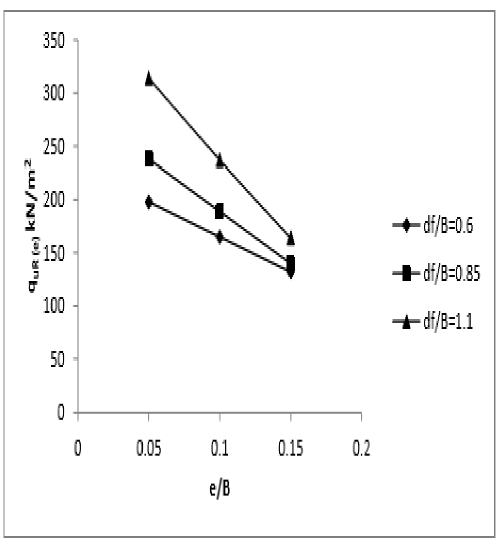

Figure 1: Variation of quR(e) with e/B for B/L=0.5

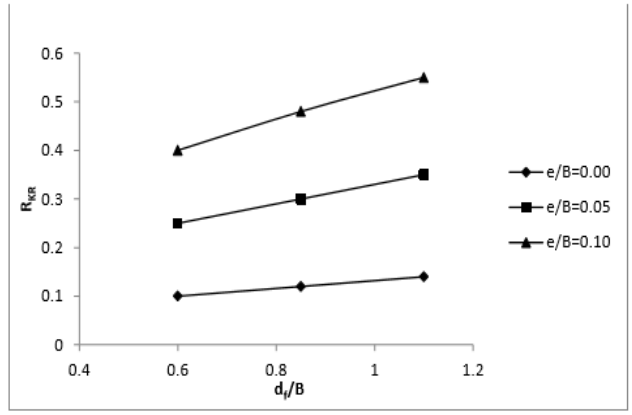

Figure 2: Variation of RKR with df/B for B/L=0.5

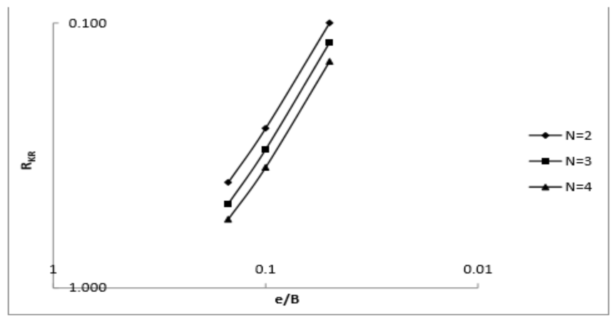

Figure 3: Variation of RKR with e/B for B/L=0.5

VII. CONCLUSIONS

A number of laboratory model tests have been conducted to determine the ultimate load bearing capacity of rectangular model footings resting over geogrid reinforced sand and subjected to vertical eccentric load. All the tests have been conducted for footing resting on the surface.

Following are the summarized results of present research work.

- The ultimate bearing capacity of the foundation for un-reinforced and reinforced soil decreases with the increase in eccentricity ratio i.e. *e/*B.
- The ultimate bearing capacity of the foundation increases with the increase in number of reinforcement layer.

- Reduction factor for the footing with B/L=0.5 & 0.33 has been derived separately and then combined to get a simple generalized equation of reduction factor for rectangular footing as shown in Equation 5.11.
- A comparison of the experiment and predicted ultimate bearing capacity for rectangular footings on reinforced sand bed by using concept of reduction factor is calculated using the derived relation. The maximum deviation of experimental from predicted is 7.14%.

The present research work is related to bearing capacity of eccentrically loaded rectangular footing with B/L = 0.5 & 0.33 resting over reinforced sand bed. Due to time constraint, other aspects related to shallow foundations could not be studied. The future work should consider the below mentioned points:

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